Evaluation of Ten Psychometric Criteria for Circumplex Structure

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This study tested for sensitivity to circumplex structure in six existing and four new psychometric criteria that assess the circumplex properties of interstitiality, equal spacing, constant radius, and no preferred rotation. Simulations showed one criterion to be sensitive to equal versus unequal axes (Fisher Test) and four to be sensitive to interstitiality versus simple structure (Gap Test, Variance Test 2, Rotation Test, and Minkowski Test). Five criteria were ineffective (Squared Loadings Index, Gap* Test, Gap Difference Test, Cosine Difference Test, and Variance Test 1). Deviation scoring improved the effectiveness of most criteria and is thus recommended for assessing circumplex structure. This study provides new and effective means for discovering complex interrelations of variables where they exist. The circumplex, which falls in the middle of a hierarchy of models in degree of parsimony, may most accurately reflect a complex domain.

Keywords: circumplex, simple structure, simulation, factor analysis

A common model for representing psychological data is simple structure (Thurstone, 1947). According to one common interpretation, data are simple structured when items or scales have non-zero factor loadings on one and only one factor (Revelle & Rocklin, 1979)1. Despite the commonplace application of simple structure, some psychological models are defined by a lack of simple structure. Circumplexes (Guttman, 1954) are one kind of model in which simple structure is lacking.

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1 This interpretation actually defines a special case of Thurstonian simple structure called independent cluster structure.
A number of elementary requirements can be teased out of the idea of circumplex structure. First, circumplex structure implies minimally that variables are interrelated; random noise does not a circumplex make. Second, circumplex structure implies that the domain in question is optimally represented by two and only two dimensions. Third, circumplex structure implies that variables do not group or clump along the two axes, as in simple structure, but rather that there are always interstitial variables between any orthogonal pair of axes (Saucier, 1992). In the ideal case, this quality will be reflected in equal spacing of variables along the circumference of the circle (Gurtman, 1994; Wiggins, Steiger, & Gaelick, 1981). Fourth, circumplex structure implies that variables have a constant radius from the center of the circle, which implies that all variables have equal communality on the two circumplex dimensions (Fisher, 1997; Gurtman, 1994). Fifth, circumplex structure implies that all rotations are equally good representations of the domain (Conte & Plutchik, 1981; Larsen & Diener, 1992).

Circumplex models have been applied to such diverse variables as cognitive abilities (Guttman, 1954); color perception (Shepard, 1962); vocational interests (Tracey & Rounds, 1993); affective states (Yik, Russell, & Feldman Barrett, 1999); interpersonal traits, problems, and goals (Acton & Revelle, 2002; Alden, Wiggins, & Pincus, 1990; Dryer & Horowitz, 1997; Gurtman & Pincus, 2000); interpersonal values (Locke, 2000); interpersonal interactions (Markey, Funder, & Ozer, 2003; Wagner, Kiesler, & Schmidt, 1995); social comparison (Locke, 2003); social support (Trobst, 2000); family relationships (Schaefer, 1997); personality disorders (Soldz, Budman, Demby, & Merry, 1993); adjectives in English and German (Saucier, Ostendorf, & Peabody, 2001); and the Big Five personality factors (Hofstee, de Raad, & Goldberg, 1992; Johnson & Ostendorf, 1993). Although a review of circumplex models applied to all of these areas is beyond the scope of this article, a recent book provides broad coverage of this domain (Plutchik & Conte, 1997). The most common method for assessing circumplex structure has been the "eyeball test"—variables are plotted in two-dimensional space using their factor loadings as coordinates and are said to comprise a circumplex if they appear to form a circle. This method has intuitive appeal but is rather unsystematic.

To bring greater rigor to bear on the question of circumplex structure, a number of authors have developed criteria that purportedly assess various circumplex properties. Joining the ranks of these authors, we present herein four new criteria that we developed to assess a relatively neglected circumplex property, lack of preferred rotation. In addition, we review six existing criteria that assess the circumplex properties of intersti-
tiality, equal spacing, and constant radius. The included criteria were selected because of the diversity of circumplex properties they are thought to assess.

Although the creation of elegant circumplex criteria has typically been regarded as the final step preceding application, it should rightly be considered only the first step; there remains the crucial task of determining which, if any, of these criteria work. One way of setting about this task is to generate simulated data sets with known properties and to determine which criteria are sensitive to various circumplex properties. In the present article, we present a simulation study designed to test the effectiveness of ten circumplex criteria. To facilitate application of the included criteria, we created a computer program called CIRC_STRUC that applies the ten criteria to an input factor matrix.

Among the criteria that have been developed to assess circumplex structure, two criteria are included without modification: Squared Loadings Index (Saucier, 1992) and Fisher Test (Fisher, 1997). The following criteria are included with the coefficient of variation (i.e., the standard deviation divided by the mean) as a new summary statistic: Gap Test (Upton & Fingleton, 1989), Gap* Test (Upton & Fingleton, 1989), Gap Difference Test (Upton & Fingleton, 1989), and Cosine Difference Test (Gurtman, 1993). In addition, the following criteria are entirely new: Variance Test 1, Variance Test 2, Rotation Test, and Minkowski Test. The following notation is used consistently in equations throughout: \( f \) denotes factors, \( nf \) denotes number of factors, \( v \) denotes vari-

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2 CIRC_STRUC is a computer program written in Pascal for the Macintosh System 7.5.5 or later. The program uses 274K of memory. The input for the program is a factor matrix saved as a text file. An opening menu allows the user to run any particular circumplex criterion or to run all ten simultaneously. The output is a tab delimited text file containing the results of applying the requested circumplex criterion to each pair of factors in the factor matrix.

3 The group of circumplex criteria included is not exhaustive. Limitations of time and resources made it impracticable to include all circumplex criteria of which we were aware. (See Tracey [2000] for an overview.) In particular, all of the included criteria are exploratory rather than confirmatory. Two criteria not included that can be used in a confirmatory way are Browne's criterion (Browne, 1992; Fabrigar, Vissar, & Browne, 1997) and Hubert and Arabie's (1987; Tracey, 1997) randomization test of hypothesized order relations. A simulation was not needed for these confirmatory criteria, because their distributional properties are known. A simulation was needed for the exploratory criteria, because they do not follow any known distribution. In addition, based on applications to empirical data, Acton and Revelle (2002) concluded that Browne's criterion is fairly similar to the Fisher Test (Fisher, 1997) in its assessment of equal axes, is more stringent than is the Gap Test (Upton & Fingleton, 1989) in its assessment of equal spacing, and does not index absence of preferred rotation at all.
ables, \( nv \) denotes number of variables\(^4\), \( \theta \) denotes angles of rotation, \( \theta_v \) denotes the angular position of a variable \( v \) on the circumplex, \( R \) denotes Minkowski \( R \), \( \phi_{fv} \) denotes the factor loading on factor \( f \) and variable \( v \), and \( \phi_{fv\theta} \) denotes the factor loading on factor \( f \) and variable \( v \) and angle of rotation \( \theta \).

**Interstitiability**

The Squared Loadings Index (SQLI; Saucier, 1992) is an index of interstitiability—that is, the degree to which variables fall in between any orthogonal pair of axes. A circumplex should have many interstitial variables, whereas a simple structure should have few. The formula for SQLI is

\[
\text{SQLI}_{fv} = \frac{\sum_{v=1}^{nv} (\phi_{1v}^2 + \phi_{2v}^2)^2}{\sum_{v=1}^{nv} (\phi_{1v}^2 - \phi_{2v}^2)^2}
\]

**Equal Spacing**

In a circumplex, variables should be uniformly distributed around the circle. This implies that they should be equally spaced. Equal spacing can be assessed in two ways: first, by calculating the angular difference of observed variables from each other; second, by calculating the angular difference of observed variables from the location where equally spaced variables should be.

*Observed gaps.* One index of equal spacing is that the distance between adjacent variables (the gaps; Upton & Fingleton, 1989) should have minimal variance. A gap is the distance from one variable to the next (so long as one is consistent for all variables, it does not matter whether the next variable is in a clockwise or counterclockwise direc-

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\(^4\) Note that, in the simulations that follow, \( nf \) was constrained to equal 2, in keeping with the definition of a circumplex. Some models, however (e.g., the abridged Big Five circumplex), contain more than two factors, thus justifying the present notation. Note also that all of the criteria imply that the center of the circumplex is identical to the point of origin of the coordinate system formed by the two factors making up the circumplex. This implies a correlation of -1 between variables at 180-degree angles from one another. Although this type of circumplex is the most commonly encountered, particularly in research on personality or emotions, it is not the only possible one. For example, correlations between variables could as well range from 0 to 1, particularly when there is a general factor, such as in research on cognitive abilities.
The Gap Test, also called *Distance to Next*, is defined as the variance of the gaps. The equation for the Gap Test is

\[
\text{Gap Test} = \sigma_{X_v}^2,
\]

where \(X_v = (\theta_{v+1} - \theta_v)\) for \(v = 1\) to \((nv - 1)\), and \(X_v = (2\pi + \theta_1 - \theta_{nv})\) for \(v = nv\).

Because gaps differ in size, an alternative index of gaps (Gap*; Upton & Fingleton, 1989) has been developed; it is the minimum gap on either side of a variable. The Gap* Test, also called *Nearest Neighbor*, is the maximum of these minima (Gaps*). The equation for the Gap* Test is

\[
\text{Gap* Test} = \max[\min(\theta_v - \theta_{v-1}, \theta_{v+1} - \theta_v)]
\]

for \(v = 2\) to \((nv - 1)\), and \(\max[\min(\theta_{nv} - \theta_{nv-1}, 2\pi + \theta_1 - \theta_{nv})]\) for \(v = nv\).

**Ideal gaps.** The Gap Difference Test (GDIFF Test), also called *Distance to Ideal*, requires the calculation of the gap between a variable and the location where an equally spaced variable should be (Upton & Fingleton, 1989). This Gap Difference (GDIFF) is calculated as the difference in radians between actual and ideal angles. The GDIFF Test is equal to the mean of the squared GDIFFs. The equation for the GDIFF Test is

\[
\text{GDIFF Test} = \frac{\sum_{v=1}^{nv} (\theta_v - \frac{2\pi v}{nv})^2}{nv}
\]

The Cosine Difference Test (CDIFF Test) is derived from the Cosine Difference (CDIFF; Gurtman, 1993), which is an item statistic that measures goodness-of-fit of a variable to a structural criterion. In the present case, the structural criterion is an equally spaced circumplex, and CDIFF measures the correlation between a variable's actual and its theoretical location on the circumplex. CDIFF is equal to the cosine of the difference in radians between actual and ideal angles. The CDIFF Test is equal to the mean of the squared CDIFFs. The equation for the CDIFF Test is

\[
\text{CDIFF Test} = \frac{\sum_{v=1}^{nv} [\cos(\theta_v - \frac{2\pi v}{nv})]^2}{nv}
\]
Constant Radius

In a true circle, the distance from the center to any point on the circle is the same. Using variables' factor loadings as coordinates in two-dimensional space, it is possible to calculate vector length for each variable. A variable's vector length on a circumplex is equal to the square root of its communality on the two circumplex dimensions. The mean vector length provides an estimate of the radius of the circle, and the standard deviation of vector lengths provides an estimate of scatter around or deviation from the circumference. The larger the standard deviation relative to the radius, the poorer the circumplex formed by a given group of variables (Fisher, 1997). Although a circumplex will ideally show constant radius, constant radius is not a sufficient condition to establish circumplex structure, because a simple structure could also show this property. The summary statistic used in all of the circumplex criteria described hereafter, including the present one, is the coefficient of variation (i.e., the standard deviation divided by the mean). The equation for the Fisher Test is

$$\text{Fisher Test} = \frac{\sigma}{X_v}, \text{ where } X_v = \sum_{f=1}^{nf} \phi_{fv}^2$$ (6)

No Preferred Rotation

A relatively neglected aspect of circumplex structure has been that there should be no preferred rotation. In a true circumplex, all rotations will be equally good representations of the domain (Conte & Plutchik, 1981; Larsen & Diener, 1992). In a simple structure, by contrast, some rotations will be better than others—not in terms of variance accounted for, which is unaffected by rotation, but in terms of such criteria as varimax, quartimax, and so on. This article presents four original circumplex criteria designed to assess whether a factor matrix has no preferred rotation: Variance Test 1, Variance Test 2, Rotation Test, and Minkowski Test.

The Variance Test 1 (VT1) is a circumplex criterion that is thought to be sensitive to rotation when there is a partial circumplex or partial simple structure—for example, when there is a circumplex on one side of the circle but no variables on the other. A partial simple structure, for example, might result from a simple structure with a large general factor that moves all variables into the first quadrant. VT1 is not thought to be sensitive to rotation when there is interstitiality versus simple structure. The values of VT1 and of the three criteria presented hereafter are computed over a range of values of
that are broken down arbitrarily into intervals such as 5 degrees. The equation for VT1 is

\[ VT_1 = \frac{\sigma_{X_\theta}}{X_0}, \text{ where } X_0 = \sigma_{Y_{\theta}^2}, \text{ and } Y_{\theta} = \frac{\phi_{1_{10}}}{\sum_{f=1}^{n_f}} \]  

(7)

The Variance Test 2 (VT2) is a circumplex criterion similar in purpose to VT1, but which is made to detect what VT1 cannot. VT2 is thought to be sensitive to rotation when there is interstitiality versus simple structure. VT2 is not, however, thought to be sensitive to rotation when there is a partial circumplex or partial simple structure—for example, when there is a circumplex on one side of the circle but no variables on the other. VT2 is calculated in the same way as VT1, except that the squared loading in VT2 replaces the loading in VT1. The equation for VT2 is

\[ VT_2 = \frac{\sigma_{X_\theta}}{X_0}, \text{ where } X_0 = \sigma_{Y_{\theta}^2}, \text{ and } Y_{\theta} = \frac{\phi_{1_{10}}^2}{\sum_{f=1}^{n_f} \phi_{f_{10}}^2} \]  

(8)

The Rotation Test (RT) is a circumplex criterion that assesses the degree to which rotation of a factor matrix makes a difference in the fit of a quartimax-like criterion—that is, makes a difference in the degree to which the coefficient of variation of a quartimax-like criterion is small. In a circumplex, rotation should make no difference; that is, all rotations should be equally good. RT measures how much a quartimax-like criterion changes as a function of angle of rotation. For each angle, RT finds the sum across variables of the variance across factors of the squared loadings. A group of variables forms a circumplex only if this sum is approximately equal for every angle. The equation for RT is

\[ RT = \frac{\sigma_{X_\theta}}{X_0}, \text{ where } X_0 = \sum_{v=1}^{m_v} \sigma_{\theta v}^2, \text{ and } \sigma_{\theta v}^2 = \frac{\sum_{f=1}^{n_f} (\phi_{f_{10}}^2 - \bar{\phi}_{10}^2)^2}{n_f - 1} \]  

(9)

Another circumplex criterion designed to assess whether a factor matrix has no preferred rotation is the Minkowski Test (MT). This test is so named because it uses a Minkowski space, which is a generalization of Euclidean space. The idea of MT is to determine whether rotating a factor matrix has any effect by determining how much distortion in the factor loadings occurs when the factors are rotated. Rotation will have
an effect for a simple structure but will have no effect for a circumplex. This is because when the Minkowski $R$ is large (e.g., $R = 100$), the biggest loading will not change noticeable with rotation.

MT requires the calculation of a Minkowski length. Whereas length in Euclidean space equals the square root of the sum of squares, length in Minkowski space equals the $R$th root of the sum of the $R$th powers. If $R = 2$, then MT is simply the communality, which is the square of the vector length; if $R > 2$, then MT is a generalization of the communality. The equation for MT is

$$MT = \frac{\sigma_{v_0}}{X_0}, \text{ where } X_0 = \sum_{v=1}^{nv} (\sum_{f=1}^{nf} \phi_{vf} R)^{2/R} \quad (10)$$

Method

Simulation Study

The research described in this section was a simulation study to test the sensitivity and specificity of the above ten criteria of circumplex structure. Sensitivity and specificity of these criteria were assessed across seven experimental conditions similar to those commonly found in real studies.

Experimental Conditions

The experimental design of this study included seven between-samples variables. Each sample was a data set having known properties with respect to the variables being studied. Each cell of the experimental design contained two samples. A total of 384 samples were generated in a 2 (raw vs. deviation scored) x 2 (unrotated vs. rotated) x 2 (equal vs. unequal axes) x 2 (interstitiality vs. simple structure) x 3 (no general factor vs. large general factor vs. variable general factor) x 2 (150 vs. 600 subjects) x 2 (64 vs. 128 variables) factorial design. The replications in each cell were generated by constructing a data matrix based on differential assignment of characteristics (manipulation of some characteristics, such as general factor size, involved differential assignment of factor loadings and factor weights, described below).

Raw scores versus deviation scores. The first variable was raw scoring versus deviation scoring. Raw scores are unmodified, whereas deviation scores are raw scores minus the mean of the subject. Deviation scoring is often used to reduce the size of the general
factor, especially if the general factor is thought to have no substantive interest (e.g., acquiescence). Note that deviation scoring is often called *ipsatizing*, but this latter term is ambiguous, because it could mean either deviation scoring or $z$-scoring. Note also that deviation scoring across variables (as used herein) should be distinguished from deviation scoring across subjects (not used).

**Unrotated versus rotated factors.** The second variable was unrotated versus rotated factors. Unrotated factors are unmodified, whereas rotated factors are rotated to some criterion. The rotation criterion used in this study was the varimax criterion. Factors were rotated by subjecting the unrotated factor matrix (obtained through principal-axis factor analysis) to an iterative procedure until the varimax criterion was maximized.

**Equal versus unequal axes.** The third variable was equal versus unequal axes. In a circle, all axes are equal in length. In an ellipse or other less regular deviation from circularity, one axis is longer than the other. In factor analytic terms, in a circle all axes account for an equal amount of variance, whereas in an ellipse one axis accounts for more variance than its orthogonal counterpart. The variable of equal versus unequal axes was manipulated by differential assignment of factor weights, described below.

**Interstitiality versus simple structure.** The fourth variable was interstitiality versus simple structure. A circumplex is minimally required to have many interstitial items—that is, items that fall in between any two orthogonal dimensions. In a simple structure, by contrast, most items fall on only one of the primary axes. The variable of interstitiality versus simple structure was manipulated by differential assignment of factor loadings, described below.

**Size of general factor.** The fifth variable was size of general factor. A general factor is a factor on which all items have a substantial loading. Three sizes of general factor weight were used (described further below): none, large, and variable. In the interpersonal literature, an instrument lacking a general factor is the Interpersonal Adjective Scales, and an instrument having a large general factor is the Inventory of Interpersonal Problems Circumplex Scales. The general factor sizes in the present study were chosen to mimic the general factor sizes in these instruments.

**Number of subjects.** The sixth variable was sample size or number of subjects. Sample size was varied to test the robustness of the circumplex criteria to sample variation. The sample sizes used were 150 and 600, representing a small and large sample, respectively.
Number of variables. The seventh variable was number of variables. Variables is a neutral term that can refer to either items or scales in a questionnaire. In the interpersonal literature, most questionnaires have either 64 or 128 items. Therefore, these were the numbers of variables used in the present study. After determining that one criterion is very sensitive to variation in number of variables, a further simulation was conducted using 8, 16, and 32 variables.

Assignment of Factor Loadings and Factor Weights

Observed scores, $X_{fv}$, were determined according to the following formula representing the common factor model, where $Z$ is a normally distributed random number, $\gamma$ is the general factor weight (see below for loadings), $\omega$ and $\xi$ are factor weights for the first and second bipolar factors, and $\varepsilon$ is the uniqueness.

$$X_{fv} = \gamma Z + \omega \phi_1 v Z + \xi \phi_2 v Z + \varepsilon v Z.$$ (11.1)

Loadings were assigned to reflect coordinates in two-dimensional space. In the interstitiality condition, such coordinates can be determined using sine and cosine curves. For example, a variable at 0 radians was assigned a loading of 1 on the first factor (x-axis) and 0 on the second factor (y-axis); a variable at 45 degrees ($\pi/4 = .79$ radians) was assigned a loading of .707 on the first factor (x-axis) and .707 on the second factor (y-axis). Loadings were determined according to the following formulas:

$$\phi_1 v = \cos(2\pi v/nv), \text{ and } \phi_2 v = \sin(2\pi v/nv).$$ (11.2)

In the simple structure condition, loadings were assigned to reflect proximity to a major axis. For example, a variable falling close to 0 radians was assigned a loading of 1 on the first factor (x-axis) and a loading of 0 on the second factor (y-axis). Loadings were determined according to the following formulas:

If $(v/nv) \geq 7/8$ or $(v/nv) < 1/8$, then $\phi_1 v = \cos(0) = 1$ and $\phi_2 v = \sin(0) = 0$.

If $(v/nv) \geq 1/8$ and $(v/nv) < 3/8$, then $\phi_1 v = \cos(\pi/2) = 0$ and $\phi_2 v = \sin(\pi/2) = 1$.

If $(v/nv) \geq 3/8$ and $(v/nv) < 5/8$, then $\phi_1 v = \cos(\pi) = -1$ and $\phi_2 v = \sin(\pi) = 0$.

If $(v/nv) \geq 5/8$ and $(v/nv) < 7/8$, then $\phi_1 v = \cos(3\pi/2) = 0$ and $\phi_2 v = \sin(3\pi/2) = -1.$ (11.3)
Factor weights were assigned differentially in six conditions, as follows. In the no general factor, equal axes condition, $\gamma = 0.0$, $\omega = 0.6$, and $\xi = 0.6$. In the no general factor, unequal axes condition, $\gamma = 0.0$, $\omega = 0.7$, and $\xi = 0.5$. In the large general factor, equal axes condition, $\gamma = 0.5$, $\omega = 0.4$, and $\xi = 0.4$. In the large general factor, unequal axes condition, $\gamma = 0.5$, $\omega = 0.4$, and $\xi = 0.3$. In the variable general factor, equal axes condition, $\gamma$ varied from 0.3 to 0.7 in increments of 0.1, $\omega = 0.4$, and $\xi = 0.4$. In the variable general factor, unequal axes condition, $\gamma$ varied from 0.3 to 0.7 in increments of 0.1, $\omega = 0.4$, and $\xi = 0.3$. In all cases, $\epsilon_v = \sqrt{1-(\phi_{1v}^2 + \phi_{2v}^2)}$.

Figure 1 shows an example of simulated raw-scored circular interstitial structure with no general factor, 64 variables, and 600 subjects. Figure 2 shows an example of simulated raw-scored ellipsoid interstitial structure with a large general factor, 64 variables, and 600 subjects. Figure 3 shows an example of simulated deviation-scored ellipsoid interstitial structure with a large general factor, 64 variables, and 600 subjects.

Figure 1. Example of simulated raw-scored circular interstitial structure with no general factor, 64 variables, and 600 subjects.
Figure 2. Example of simulated raw-scored ellipsoid interstitial structure with a large general factor, 64 variables, and 600 subjects.

Figure 3. Example of simulated deviation-scored ellipsoid interstitial structure with a large general factor, 64 variables, and 600 subjects.

Statistical Analyses

The statistical analyses performed were analyses of variance (ANOVAs). The independent variables were the seven experimental conditions described above and their interactions. The dependent variables were scores on each circumplex criterion when ap-
plied to the first pair of factors extracted using principal-axis factor analysis without rotation.

Note that the approach was “blind” as to whether the first pair of factors were indeed the circumplex factors, and, particularly in the large general factor condition, they were not. The reason for this seemingly paradoxical approach was fidelity to real-life applications, in which the primary use of the circumplex criteria is to determine the presence or absence of circumplex structure when this is unknown at the outset. Thus, we intentionally simulated conditions (e.g., the large general factor condition) in which the application of criteria based on angular positions on the first pair of factors (Gap Test, Gap* Test, GDPFF Test, and CDIFF Test) would be inadequate, with the expectation that we would find that they were inadequate in these conditions.

**Hypotheses**

The hypotheses, broadly speaking, were that the circumplex criteria would work. That is, under all conditions, each circumplex criterion should contribute some valuable information, either whether something is an interstitial structure versus simple structure, or whether something has equal versus unequal axes. The following circumplex criteria were expected to be sensitive to interstitiality versus simple structure: SQLI, Gap Test, Gap* Test, GDPFF Test, CDIFF Test, RT, VT2, and MT. Only the Fisher Test and VT1 were expected to be sensitive to equal versus unequal axes.

**Results**

Three of the four criteria (VT2, RT, and MT) that assess the property of no preferred rotation were highly correlated (Table 1). In the case of RT and MT, the correlation was actually .99. This should not be taken to indicate, however, that all of the criteria were redundant. In particular, among criteria that work, the Fisher Test seemed to be an isolated case, assessing a property, constant radius, different from all the rest. Similarly, except for its correlation with VT1, the Gap Test seemed to assess a property, equal spacing, that was fairly distinctive. The existence of separate clusters of criteria is encouraging, because special distinctive. The existence of separate clusters of criteria is encouraging, because special weight is to be attached to conclusions based on unlike criteria when they converge on the same result.

The ANOVAs in the simulation resulted in a number of main effects and interactions. The effects of greatest interest involved equal versus unequal axes and interstitial-
ity versus simple structure (Table 2). A circumplex criterion was said to “work” if an effect involving either of these two manipulations was of non-negligible size and was statistically significant at the $p < .0001$ level, otherwise not. The most consistently observed interactions were those involving one of these two manipulations and deviation scoring (Table 2). As will be seen, deviation scoring was a critical variable in determining if and when a circumplex criterion worked. The full ANOVAs for each criterion are available from the first author.

Table 1

**Intercorrelations of Criteria**

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<td>0.01</td>
<td>0.16</td>
<td>0.07</td>
<td>0.23</td>
<td>0.22</td>
<td>0.85</td>
<td>0.99</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

**Criteria That Do Not Work**

*Squared Loadings Index.* SQLI was sensitive to neither interstitiality nor equal axes. Thus, SQLI had distinct problems that make it unusable as a circumplex criterion.

*Gap* Test. The Gap* Test was sensitive to neither interstitiality nor equal axes. Indeed, when sample size was large ($N = 600$), the Gap* Test was lower for simple structures than for interstitial structures, $F(1, 192) = 20.0, \eta^2 = .01$, whereas it should have been higher. Thus, the Gap* Test is unusable as a circumplex criterion.
**GDIFF Test.** The GDIFF Test was sensitive to neither interstitiality nor equal axes. Thus, the GDIFF Test should not be used as a circumplex criterion.

**CDIFF Test.** Like the GDIFF Test, the CDIFF Test was sensitive to neither interstitiality nor equal axes. Thus, the CDIFF Test too should be avoided as a circumplex criterion.

**Variance Test 1.** VT1 yielded a statistically significant effect of negligible size for detecting equal axes, \(F(1, 192) = 12.4, \eta^2 = .00\). Moreover, in deviation scored data, VT1 sometimes rendered results in the opposite of the correct direction. Therefore, VT1 probably should be avoided as a circumplex criterion.

Table 2

<table>
<thead>
<tr>
<th>Criterion</th>
<th>EU</th>
<th>EU x Dev</th>
<th>IS</th>
<th>IS x Dev</th>
<th>Multiple (R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SQLI</td>
<td>7.0 (.02)</td>
<td>0.4 (.00)</td>
<td>0.5 (.00)</td>
<td>0.1 (.00)</td>
<td>.50</td>
</tr>
<tr>
<td>Gap</td>
<td>4.7 (.00)</td>
<td>40.5** (.00)</td>
<td>1,038.8** (.03)</td>
<td>83.1 (.00)</td>
<td>.99</td>
</tr>
<tr>
<td>Gap*</td>
<td>9.3 (.01)</td>
<td>12.5 (.02)</td>
<td>8.1 (.01)</td>
<td>3.4 (.00)</td>
<td>.73</td>
</tr>
<tr>
<td>GDIFF</td>
<td>0.2 (.00)</td>
<td>0.1 (.00)</td>
<td>5.2 (.00)</td>
<td>1.0 (.00)</td>
<td>.68</td>
</tr>
<tr>
<td>CDIFF</td>
<td>0.6 (.00)</td>
<td>0.2 (.00)</td>
<td>2.7 (.00)</td>
<td>0.4 (.00)</td>
<td>.65</td>
</tr>
<tr>
<td>Fisher</td>
<td>270.5** (.08)</td>
<td>143.1** (.04)</td>
<td>1.1 (.00)</td>
<td>24.6** (r) (.01)</td>
<td>.95</td>
</tr>
<tr>
<td>VT1</td>
<td>111.0** (.00)</td>
<td>12.2 (.00)</td>
<td>12.4 (.00)</td>
<td>99.5** (r) (.00)</td>
<td>.98</td>
</tr>
<tr>
<td>VT2</td>
<td>24.6** (.00)</td>
<td>17.6** (.00)</td>
<td>3,869.7** (.17)</td>
<td>784.6** (.03)</td>
<td>.98</td>
</tr>
<tr>
<td>RT</td>
<td>4.4 (.00)</td>
<td>2.7 (.00)</td>
<td>5,261.1** (.23)</td>
<td>1,265.6** (.05)</td>
<td>.98</td>
</tr>
<tr>
<td>MT</td>
<td>3.7 (.00)</td>
<td>1.2 (.00)</td>
<td>6,454.8** (.55)</td>
<td>1,262.5** (.11)</td>
<td>.98</td>
</tr>
</tbody>
</table>

*Note. \(F(1, 192)\) appears first, followed in parentheses by the eta-squared (\(\eta^2\)) correlation coefficient. Multiple \(R^2\) applies to the full model made up of all main effects and all higher-order interactions. EU = equal axes versus unequal axes. IS = interstitiality versus simple structure. Dev = deviation scoring versus raw scoring. ** = statistically significant at \(p < .0001\). (r) = reverse direction of correct results.

**Criteria That Work**

The following circumplex criteria worked under some experimental conditions: Fisher Test, Gap Test, Variance Test 2, Rotation Test, and Minkowski Test. To say that they worked is to say that they were sensitive to equal (versus unequal) axes or to interstitial (versus simple) structure under some conditions.
Figures 1 to 5 represent the relative frequency distributions of the criteria under several specified conditions. For example, the distributions of a given criterion for deviation scored data can be compared when they arose from an underlying simple structure and when they arose from underlying interstitiality. Although there may be overlap in these two distributions, the hope is that the overlap will be small so that, at some value, one can be highly certain that one has found interstitiality rather than simple structure. Thus, the ideal distribution would be one in which, for example, the relative frequency of simple structures reached 1.0 at a value of the criterion lower than that at which the relative frequency of interstitiality rose above 0.0—an ideal that is indeed approximated for some criteria.

*Fisher Test.* The Fisher Test was hypothesized to be sensitive to equal axes but not to interstitiality. This was indeed what was found. The main effect of equal axes was of reasonable size, \( F(1, 192) = 270.5, \eta^2 = .08 \), whereas the main effect of interstitiality was of negligible size and was not statistically significant. Deviation scoring made the Fisher Test more sensitive to equal axes than did raw scoring, \( F(1, 192) = 143.1, \eta^2 = .04 \). With regard to interstitiality, the only statistically significant effect was that the Fisher Test actually gave the reverse of the correct result when deviation scored, \( F(1, 192) = 24.6, \eta^2 = .01 \). Thus, although the Fisher Test was a good index of equal axes, it is not to be used as an index of interstitiality.

![Cumulative probabilities of Fisher Test frequencies](Figure 4. Cumulative probabilities of Fisher Test frequencies in raw (Raw) versus deviation scored (Dev) data by equal axes (E) versus unequal axes (U) conditions.)
A value of Fisher Test lower than .10 almost always indicated equal axes. In either raw scored or deviation scored data, a value of .15 was about twice as likely to indicate equal as unequal axes, whereas a value of .21 was equally likely to indicate equal as unequal axes (Figure 4). In deviation scored data, equal axes had an appreciably lower value than unequal axes up to about .40, after which both had approximately equal values. Because the Fisher Test was better at distinguishing equal from unequal axes in deviation scored data than in raw scored data, deviation scoring is recommended when using the Fisher Test for this purpose.

**Gap Test.** The Gap Test was hypothesized to be sensitive to interstitiality but not to equal axes. In fact, this was exactly what was found; the Gap Test was somewhat sensitive to interstitiality, $F(1, 192) = 1,038.8$, $\eta^2 = .03$, but not to equal axes, $F(1, 192) = 4.7$, $ns$, $\eta^2 = .00$.

![Cumulative probabilities of Gap Test frequencies in raw (Raw) versus deviation scored (Dev) data by interstitiality (I) versus simple structure (S) conditions.](image)

**Figure 5.** Cumulative probabilities of Gap Test frequencies in raw (Raw) versus deviation scored (Dev) data by interstitiality (I) versus simple structure (S) conditions.

In raw scored data, a Gap Test value less than .01 almost certainly indicated interstitiality; a value less than .04 was at least twice as likely to indicate interstitiality as simple structure; and throughout its range, the Gap Test was likely to be lower for interstitiality than for simple structure (Figure 5). In deviation scored data, a Gap Test value less than .03 almost certainly indicated interstitiality; a value less than .05 was at least twice as likely to indicate interstitiality as simple structure; and throughout its
range, the Gap Test was likely to be lower for interstitiality than for simple structure. Thus, the Gap Test was a useful method for distinguishing interstitiality from simple structure.

The effect of number of variables on the Gap Test was substantial, $F(1, 192) = 3,458.4, \eta^2 = .11$. This was because the Gap Test is a variance of the gaps, and although the variance of an eight-variable circumplex will be the same as that of a 128-variable circumplex—namely, zero—the variance of an eight-variable simple structure will be substantially larger than that of a 128-variable simple structure, because many variables clumping together have a small variance. The large effect of number of variables on the Gap Test necessitated the addition of a further simulation using 8, 16, and 32 variables (in addition to 64 and 128). The Gap Test was the only criterion to necessitate such treatment as a result of a sizable number-of-variables effect.

**Variance Test 2.** VT2 was hypothesized to be sensitive to interstitiality but not to equal axes. Indeed, VT2 was sensitive to interstitiality, $F(1, 192) = 3,869.7, \eta^2 = .17$, but not to equal axes, $F(1, 192) = 24.5, \eta^2 = .00$. Deviation scoring made VT2 more sensitive to interstitiality, $F(1, 192) = 784.6, \eta^2 = .03$.

![Figure 6. Cumulative probabilities of Variance Test 2 frequencies in raw (Raw) versus deviation scored (Dev) data by interstitiality (I) versus simple structure (S) conditions.](image-url)
In raw scored data, a VT2 value less than .25 almost certainly indicated interstitiality; a value less than .30 was at least twice as likely to indicate interstitiality as simple structure; and throughout its range, VT2 was likely to be lower for interstitiality than for simple structure (Figure 6). In deviation scored data, a VT2 value less than .40 almost certainly indicated interstitiality; a value less than .58 was at least three times as likely to indicate interstitiality as simple structure; a value less than .65 was at least twice as likely to indicate interstitiality as simple structure; and throughout its range, VT2 was likely to be lower for interstitiality than for simple structure (Figure 6). Thus, VT2 was a powerful method for distinguishing interstitiality from simple structure, particularly when data were deviation scored.

VT2 was particularly sensitive to interstitiality in deviation scored data in which there was no general factor, \( F(2, 192) = 298.6, \eta^2 = .03 \). In raw scored data that had a large general factor or variable general factor, VT2 was insensitive to interstitiality (although the trend was in the incorrect direction). Thus, VT2 is best used with data in which there is no general factor. In every case, deviation scoring is strongly recommended.\(^5\)

Rotation Test. RT is hypothesized to be sensitive to interstitiality but not to equal axes. This was indeed what was found. The main effect of interstitiality was large, \( F(1, 192) = 5,261.1, \eta^2 = .23 \), whereas the main effect of equal axes was negligible and was not statistically significant. Although deviation scoring had no effect on the detection of equal axes, it caused a pronounced improvement in the detection of interstitiality, \( F(1, 192) = 1,265.6, \eta^2 = .05 \).

In raw scored data, an RT value less than .04 almost certainly indicated interstitiality; a value less than .09 was at least twice as likely to indicate interstitiality as simple structure; and throughout its range, RT was likely to be lower for interstitiality than for simple structure (Figure 7). In deviation scored data, an RT value less than .14 almost certainly indicated interstitiality; a value less than .31 was at least twice as likely to indicate interstitiality as simple structure; and throughout its range, RT was likely to be lower for interstitiality than for simple structure (Figure 7). Thus, RT was a powerful method for distinguishing interstitiality from simple structure, particularly when data were deviation scored.

\(^5\) Because VT2 was an effective circumplex criterion, whereas VT1 was not, it is recommended that in future applications VT2 be referred to simply as the Variance Test or VT.
Figure 7. Cumulative probabilities of Rotation Test frequencies in raw (Raw) versus deviation scored (Dev) data by interstitiality (I) versus simple structure (S) conditions.

Minkowski Test. MT was hypothesized to be sensitive to interstitiality but not to equal axes. This was indeed what was found. The main effect of interstitiality was large, \( F(1, 192) = 6,454.8, \eta^2 = .55 \), whereas the main effect of equal axes was trivial and was not statistically significant. Although deviation scoring had no effect on the detection of equal axes, it caused a pronounced improvement in the detection of interstitiality, \( F(1, 192) = 1,265.5, \eta^2 = .11 \).

In raw scored data, an MT value less than .03 almost certainly indicated interstitiality; a value lower than .05 was at least twice as likely to indicate interstitiality as simple structure; and throughout its range, MT was likely to be lower for interstitiality than for simple structure (Figure 8). In deviation scored data, an RT value less than .06 almost certainly indicated interstitiality; a value less than .16 was at least twice as likely to indicate interstitiality as simple structure; and throughout its range, MT was likely to be lower for interstitiality than for simple structure (Figure 8). Thus, MT was a powerful method for distinguishing interstitiality from simple structure, particularly when data were deviation scored.
MT was particularly sensitive to interstitiality in deviation scored data with large sample sizes (e.g., $N = 600$), $F(1, 192) = 65.5$, $\eta^2 = .01$. In raw scored data that had a large general factor or variable general factor, MT was insensitive to interstitiality (although the effect was in the incorrect direction), $F(2, 192) = 364.6$, $\eta^2 = .06$. Thus, MT is best used in data with large sample sizes. Where there is a general factor, the data must be deviation scored to yield correct results. Deviation scoring is strongly recommended in every case.

**Discussion**

The sensitivity and specificity of ten criteria for circumplex structure were tested on simulated data sets. Five criteria contributed some valuable information, either allowing one to reject unequal axes in favor of equal axes (Fisher Test) or allowing one to reject simple structure in favor of circumplex structure (Gap Test, Variance Test 2, Rotation Test, Minkowski Test).
Visible Versus Invisible Conditions

The interaction of equal versus unequal axes with interstitiality versus simple structure, which is common for tests that work, would require some theoretical explanation if it were of appreciable size ($\eta^2$ ranges from .00 to .01), because it involves only factors that are determined by the tests, not by the investigator. Combinations of "invisible" conditions (equal versus unequal axes, interstitiality versus simple structure, or general factor) and "visible" conditions (e.g., deviation scoring or rotation) may require some theoretical explanation, but they are also of practical value in manipulating real data.

For example, a visible property that can be manipulated is deviation scoring. Investigators can easily use the information that deviation scoring is beneficial for manipulating and interpreting their data. This is unlike finding some interaction between equal versus unequal axes and interstitiality versus simple structure, because these are not manipulable but are hidden properties that investigators want to discover.

Deviation Scoring: Boon or Blight?

With respect to the visible properties, the goal is to make some sort of practical recommendation for manipulating and interpreting data—if possible, one that applies generally to all of the criteria that work. A finding that generalizes across most criteria is that deviation scoring makes criteria better at doing what they were created to do—although it cannot enable them to do what they were not created to do. Deviation scoring works because it removes a general factor if there is one and has little effect if there is not. If the data are perfectly circumplexical, then the sum of all items is zero and deviation scoring will have no effect. If the data have a general factor, then the sum of all items is greater than zero and deviation scoring will reduce the general factor.

Deviation scoring can actually prevent the Variance Test 2 from rendering incorrect results. When raw scored data have a large general factor or a variable general factor, VT2 will mislabel a simple structure as an interstitial structure. Because deviation scoring reduces a general factor, it allows VT2 to render the results it would otherwise render in raw scored data with no general factor, namely a correct identification of interstitiality versus simple structure.

The Gap Test is hypothesized to be able to detect interstitiality; although it can do this without deviation scoring, it can do it even better with deviation scoring. It is not hypothesized to be able to detect equal axes, nor can it; deviation scoring in some cases actually causes it to register the opposite of the correct result (although these are weak
trends). Thus, deviation scoring is a mixed blessing: Used with the correct interpretation, it can enhance the power of a test; used with an incorrect interpretation, it can render fallacious results.

There are many three-way interactions involving deviation scoring. In some, deviation scoring makes a good test better. In others (such as interstitiality x deviation scoring x general factor in VT2, discussed above), deviation scoring makes a bad test good. In the interest of keeping things simple, we have looked at the main effects of equal axes and interstitiality and the two-way interactions of equal axes x deviation scoring and interstitiality x deviation scoring. Those that are of non-negligible size and are statistically significant (using a stringent alpha level of .0001) warrant further consideration; the others (SQLI, Gap*, GDIFF, CDIFF, and VT1) can be ignored.

The data presented herein provide ample documentation for a broad conclusion regarding deviation scoring: Used properly, deviation scoring is a circumplex researcher's best friend. Deviation scoring is especially beneficial if one has no way of knowing a priori whether the data contain a general factor, which may be typical of real-life circumstances. This is an important qualification, because all of the analyses presented herein are predicated on the assumption that one does not know whether or not the data have a general factor. Thus, the criteria that are said to work are useful primarily for exploratory purposes.

A Hierarchy of Models

A circumplex falls somewhere in the middle of a hierarchy of models. The hierarchy, in order of descending parsimony, is as follows:

(a) one factor;
(b) multiple factors but simple structure;
(c) multiple factors with some interstitial variables;

---

When there is a general factor that is not removed through deviation scoring or some other procedure, rotation will have the effect of eliminating a circumplex where one exists. Nevertheless, we have called a condition characterized by a circumplex plus a general factor a circumplex condition, even when rotated, because a researcher normally cannot know a priori whether there is a general factor in the data. The general factor condition is considered an invisible condition because, although some general factors can be identified as the first extracted factor, on which all variables have positive loadings, other, more subtle general factors simply elevate bipolar factor loadings and therefore are not easily detectable.
(d) multiple factors and variables show equal spacing around pairs of factors;
(e) multiple factors and all items are best described in terms of more than two factors.

Model d (which includes the circumplex) falls somewhere between model c and model e in degree of parsimony. If an instrument can be characterized by models a, b, or e, then it is not a circumplex. Having multiple factors with some interstitial variables (model c) is a necessary but not sufficient condition for an instrument to be a circumplex. In a circumplex (model d) the variables must also be equally spaced.

Although a circumplex is less parsimonious than a two-dimensional simple structure or a one-dimensional test, it may also be a more accurate reflection of a complex field. It would be easy to devise a simple structure made up of variables measuring two unrelated domains, such as hardness and brightness. It is much more difficult to devise a circumplex structure, because one must sample from a complex domain made up of interrelated variables. The latter, although less parsimonious, would be a more significant finding than the former.

Examples of complex domains are affective states and interpersonal traits. It is possible to feel emotions that are subtle combinations of valence and arousal; for example, excitement is a positive emotion that also reflects high arousal. It is also possible to behave toward others in ways that are subtle combinations of dominance and friendliness; for example, gregariousness is a friendly trait that also reflects high dominance. In each case, the particular item is drawn from a complex domain comprised of interrelated subdomains. Where such interrelationships exist, it is desirable that they be discovered. The present study has provided new and effective means of doing so.

Limitations

The simulations presented herein are based on idealized conditions. An attempt was made, however, to represent not only a diversity of idealized conditions, but also conditions that actually appear in practice. Thus, although the application of these results may be limited to cases similar to those included in the simulation, such cases do occur.

The differences between the interstitiality versus simple structure conditions and the equal versus unequal radius conditions were designed to be large enough that one could distinguish them by inspection of the plot of variables in the factor space. Several criteria failed this evaluation, and these failures may be as revealing as the successes. Nevertheless, although further research on the criteria that worked seems warranted, these criteria can be recommended as having passed their first important test.
References


